 1880

Solutions	Marks	Remarks
1. (a) Loss per coin = 3000 − 2700 = \$300	1A	
The total loss = 300 x 10		
= \$3000	1A	
	2	
(b) The percentage loss = $\left(\frac{3000}{30000} \times 100 \right)\%$	1M	免較亦可
= 10%	1A	
	2	
2. (a) $\frac{a}{\sqrt{a}} = \frac{a}{a^{\frac{1}{2}}}$	1A (optional)	OR $\sqrt{a}$
$= a^{1-\frac{1}{2}}$		
$= a^{\frac{1}{2}}$	1A	Do not accept $\sqrt{a}$
	2	
(b) $\frac{\log(a^2) + \log(b^4)}{\log(ab^2)} = \frac{2\log a + 4\log b}{\log a + 2\log b}$	1M+1M	1M for $\log p^n = n\log p$ 1M for $\log pq = \log p + \log q$
$= 2$	1A	
	3	
<u>Alternatively</u>		
$\frac{\log(a^2) + \log(b^4)}{\log(ab^2)} = \frac{\log(a^2b^4)}{\log(ab^2)}$	1M	
$= \frac{2\log(ab^2)}{\log(ab^2)}$	1M	
$= 2$	1A	
3. $\frac{\sin^2\theta}{\cos\theta} = \frac{-3}{2}$	1A	
$\frac{1-\cos^2\theta}{\cos\theta} = \frac{-3}{2}$	1M	For $\sin^2\theta = 1 - \cos^2\theta$
$2\cos^2\theta - 3\cos\theta - 2 = 0$	1A	
$(2\cos\theta + 1)(\cos\theta - 2) = 0$		
$\cos\theta = -\frac{1}{2}$ (as $\cos\theta \neq 2$)	1A	
$\therefore\ \theta = 120°$ or $240°$ ($\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ OR 2.09 or 4.19)	1A+1A	
	6	

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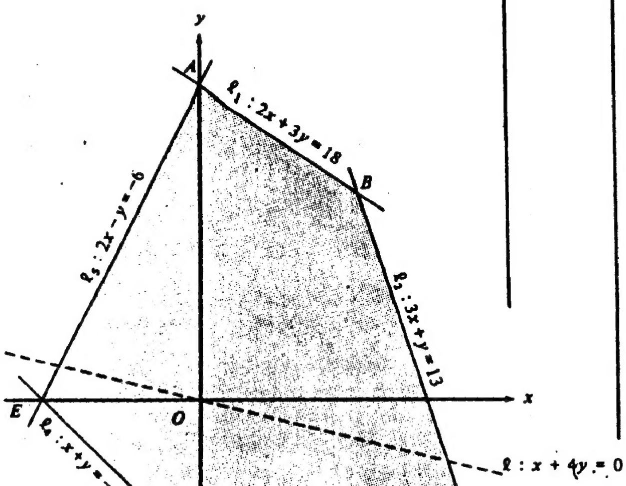
Solutions	Marks	Remarks
4. (a) (i) $6x + 1 \geqslant 2x - 3$		
$6x - 2x \geqslant -3 - 1$	1M	Collecting terms
$\therefore\ x \geqslant -1$	1A	
(ii) $(2 - x)(x + 3) > 0$		OR $(+) \times (+)$ $-3 < x < 2$ 1A $(-) \times (-)$ no solution $\therefore -3 < x < 2$ 1A Accept graphical representation of solution. Withhold 1 mark for weak inequality.
(By considering the graph of the quadratic function), the solution is given by $-3 < x < 2$.	2A	
(b) From (i) and (ii), the values of x are given by		
$-1 \leqslant x < 2$.	2A	1 mark for $-1 \leqslant x \leqslant 2$, etc
	6	
5. By sliding the line ℓ, it is observed that p takes the greatest value at A	1	
and the least value at D.	1	
Putting $x = 0$ in ℓ_1, $y = 6$		
$\therefore\ A = (0, 6)$	1A	
The greatest value of $P = 22$.	1A	
Putting $y = -2$ in ℓ_4, $x = -1$		
$D = (-1, -2)$.	1A	
The least value of $P = -11$.	1A	
	6	
<u>Alternatively</u>		
$A = (0, 6)$, $B = (3, 4)$, $C = (5, -2)$, $D = (-1, -2)$, $E = (-3, 0)$ ...	1A+1A	1A for any two correct points
The values of P at these points are respectively 22, 17, -5, -11, -5	1M+ 1A+ 1A	Testing value at any pt. 1A for any one correct value
$\therefore$ P takes the greatest value of 22 at A and the least value of -11 at D.	1A	Must first score the above 5 points



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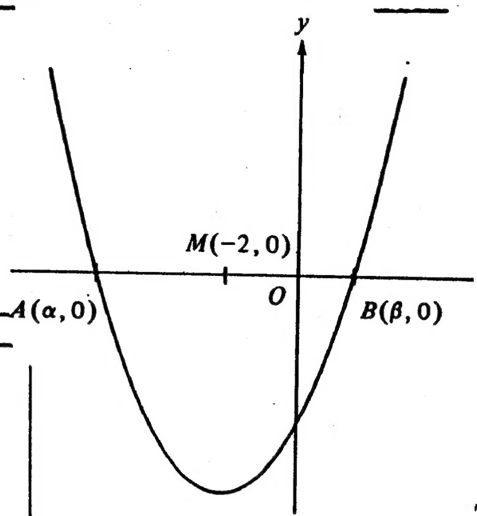
Solutions	Marks	Remarks
6. (a) α and β are the roots of $x^2 + px + q = 0$.		
$\therefore \alpha + \beta = -p$	1A	
M(−2, 0) is the mid-point of AB		
$\therefore \dfrac{\alpha+\beta}{2} = -2$	1A	
$p = 4$	1A 3	
(b) Now $\alpha\beta = q$	1A	
$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$		
$(-4)^2 = 26 + 2q$	1M	Formula correct
$\therefore q = \dfrac{16 - 26}{2}$		
$= -5$	1A 3	
7. (a) The remainder is $(-1)^{1000} + 6$	1M	Optional
$= 7$	1A 2	
(b) (i) Putting $x = 8$,	1M	optional, or quoting result in (a)
by (a), the remainder is 7	1A	
(ii) The remainder of 8^{1000} when divided by 9 is $7 - 6$	1M	optional
$= 1$	1A 6	

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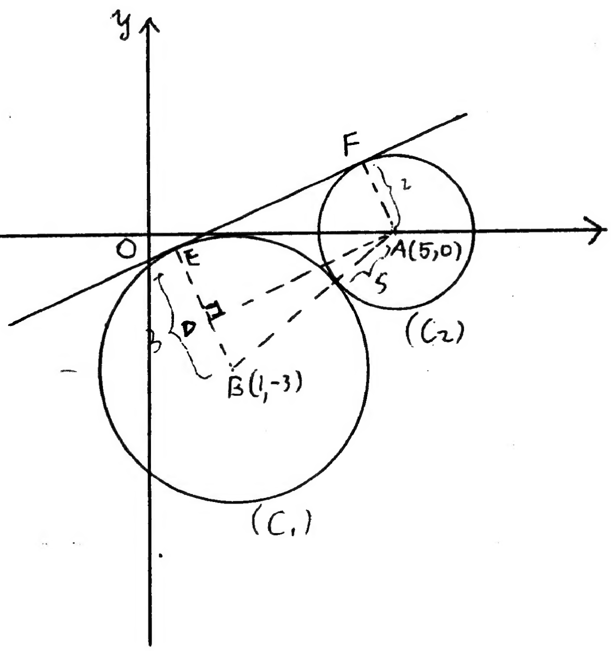
Solutions	Marks	Remarks
8. (a) Centre = (1, −3)	1A	$x=1,\ y=-3$
Radius = $\sqrt{(-1)^2+(3)^2-1}=3$	1A	
	2	
(b) Distance between the centre and A		
$=\sqrt{(5-1)^2+(0--3)^2}$	1M	
= 5	1A	
> radius of (C_1) (=3))		
$\therefore$ A lies outside (C_1))...	1M	
	3	
(c) (i) s = 5 − 3	1M	
= 2	1A	2
(ii) Equation of (C_2) is $(x-5)^2+(y-0)^2=2^2$	1A	
or $x^2+y^2-10x+21=0$		
	3	
(d)	1	For sketch. A line touching two circles at 2 distinct points. May draw the other common tangent. Follow through.
		
EF = DA)		
BD = BE − AF)		
$EF=\sqrt{AB^2-BD^2}$		
$=\sqrt{5^2-(3-2)^2}$	1M+1A	
$=\sqrt{24}$	1A	
$=2\sqrt{6}$ (= 4.90)		Any figure roundable to 4.90
	4	

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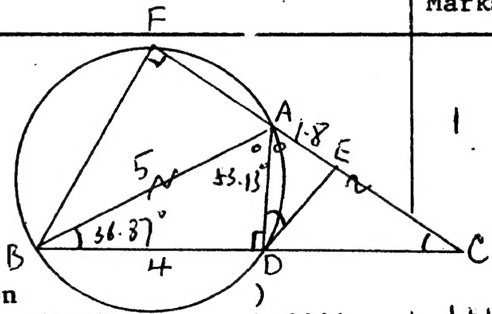
Solutions	Marks	Remarks
9.(a) Consider $\triangle$s ABD and ACD.		
As AB is a diameter, $\angle ADB = 90°$.	1	
As BDC is a straight line, $\angle ADC = 90°$		
As AB = AC and AD is common $\triangle ABD$ and $\triangle ACD$ are congruent (RHS)	1+1	1 for AD=AD / AB=AC / $\angle ABD = \angle ACD$. 1 for correct reasoning
	3	
(b) Consider $\triangle$s ABD and ADE		
$\because \triangle ABD \cong \triangle ACD$		
$\angle BAD = \angle CAD$	1	
Since DE is a tangent, $\angle ADE = \angle ABD$ ($\angle$ in alt. seg.)	1	
$\therefore \triangle ABD \sim \triangle ADE$		
	2	
(c) (i) As $\angle ADB = 90°$		OR
$AD = \sqrt{AB^2 - BD^2}$		
$= \sqrt{5^2 - 4^2} = 3$	1A	AD = 3 .... 1A
$\because \triangle ABD \sim \triangle ADE$		$\angle ABD = \cos^{-1} 0.8$ 1M
$\frac{DE}{3} = \frac{4}{5}$ $\frac{AB}{AD} = \frac{BD}{DE}$, $\frac{5}{3} = \frac{4}{DE}$	1M	DE = 3cos36.87°
$\therefore DE = 2\frac{2}{5}$ (= 2.4)	1A	= 2.4 .... 1A
(ii) Consider $\triangle$s BCF and ABD		
$\because$ AB is a diameter, $\angle AFB = 90°$ $= \angle ADB$	1	
As $\angle BCF = \angle ABD$,		
$\triangle$s BCF and ABD are similar	1	
$\frac{AF + 5}{8} = \frac{4}{5}$	1A	
$AF = 1\frac{2}{5}$ (= 1.4)	1A	
	7	
Alternatively		
(a) $\because$ AB is a diameter, $\angle ADB = 90°$ $\because$ ABC is an isosceles triangle and $BC \perp AD$	1A	May also use AAS
$\triangle ABD \cong \triangle ACD$	1+1	
(c)(ii) (1) $\angle ACB = \angle ABC = 36.87°$	1A	Accept 36.9°
$\angle AFB = 90°$	1	optional
$\frac{AF + 5}{8} = \cos 36.9°$	1A	
AF = 1.40	1A	
(2) $\angle ABC = \angle ACB = 36.87°$	1A	Accept 36.9°
$\angle AFB = 90°$	1	optional
$\angle BAF = \angle ABC + \angle ACB = 73.7°$		
$\cos 73.7° = \frac{AF}{5}$	1A	
AF = 1.40	1A	



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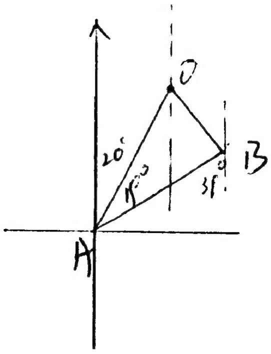
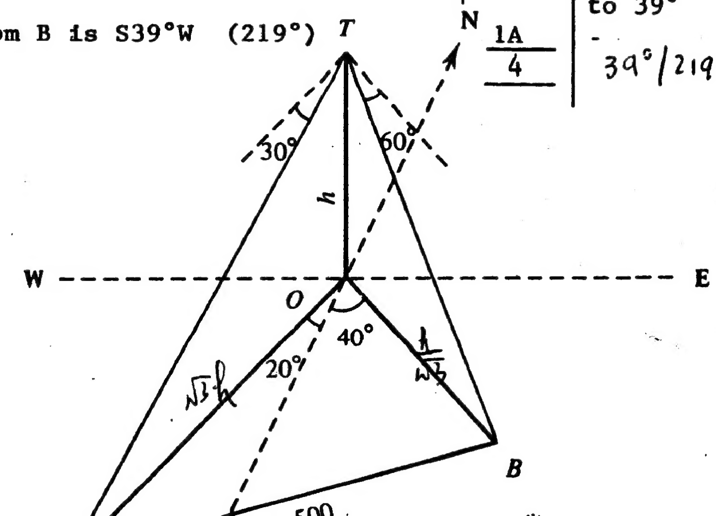
Solutions	Marks	Remarks
10.(a) $\frac{OT}{OA} = \tan 30°$ $\left(\frac{CA}{CT} = \tan 60°\right)$	1A	2 + 1
$\therefore$ OA $= \frac{h}{\tan 30°}$	1A	
$= h\sqrt{3}$ metres $(= 1.73h)$		
Similarly OB $= \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}$ metres $(= 0.577h)$	1A	
	3	
(b) $\angle AOB = 60°$	1A	
By the cosine rule,		
$AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos\angle AOB$		
$= (h\sqrt{3})^2 + \left(\frac{h}{\sqrt{3}}\right)^2 - 2(h\sqrt{3})\left(\frac{h}{\sqrt{3}}\right)\cos 60°$	1M	
$= 3h^2 + \frac{h^2}{3} - h^2$		
$= \frac{7}{3}h^2$		
$\therefore$ AB $= h\sqrt{\frac{7}{3}}$ metres $(1.53h)$	1A	Any fig. roundable to 1.53h
As $h\sqrt{\frac{7}{3}} = 500$	1M	
$h = 500\sqrt{\frac{3}{7}}$ $(= 327$ or $328)$	1A	Any figure roundable to 327 or 328
	5	
(c) By the sine rule $\frac{h/\sqrt{3}}{\sin\angle OAB} = \frac{500}{\sin 60°}$	1M	
$\sin\angle OAB = \frac{h}{\sqrt{3}} \times \frac{\sin 60°}{500}$		
$= \frac{500\sqrt{\frac{3}{7}}}{\sqrt{3}} \times \frac{\frac{\sqrt{3}}{2}}{500} = \frac{1}{2}\sqrt{\frac{3}{7}}$ (0.327)		
$\therefore \angle OAB = 19.1° = 19°$ (correct to the nearest degree)	1A	
(i) The bearing of B from A is N39°E (039° or 39°)	1A	Accept figure roundable to 39°
(ii) The bearing of A from B is S39°W (219°)	1A	39°/219°
	4	



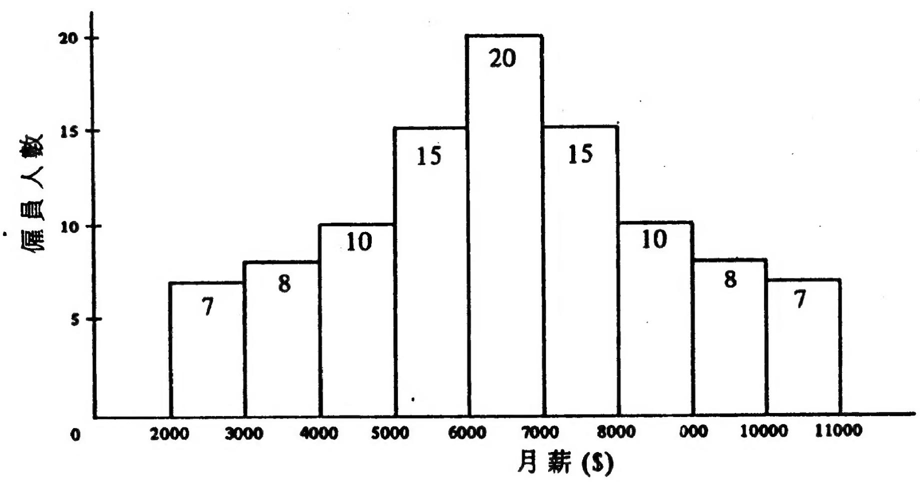


Solutions	Marks	Remarks
11.(a) (i) $S = 2\pi r^2 + 2\pi rh$	1A	
(ii) As $V = \pi r^2 h$, $h = \dfrac{V}{\pi r^2}$		OR
$S = 2\pi r^2 + 2\pi r\left(\dfrac{V}{\pi r^2}\right)$	1M	$2\pi r^2 + \dfrac{2V}{r} = 2\pi r^2 + \dfrac{2(\pi r^2 h)}{r}$
$= 2\pi r^2 + \dfrac{2V}{r}$	1	$= S$
	3	
(b) Putting $V = 2\pi$, $S = 6\pi$		
$6\pi = 2\pi r^2 + \dfrac{2(2\pi)}{r}$		
$\therefore\ r^3 - 3r + 2 = 0$	1	
By inspection, $r = 1$ is a root (or $r = -2$)	1A	OR $r-1$ is a factor OR $r+2$ is a factor
$\therefore\ r^3 - 3r + 2 = (r-1)(r^2 + r - 2)$		
$= (r-1)^2(r+2)$	1A	
$= 0$		
i.e. $r = 1$ (as $r \neq -2$)	1A	
	4	
(c) Putting $V = 3\pi$, $S = 10\pi$, we have		
$10\pi = 2\pi r^2 + \dfrac{2(3\pi)}{r}$		
$r^3 - 5r + 3 = 0$	1A	
Let $f(r) = r^3 - 5r + 3$		
$f(1) < 0$ and $f(2) > 0$, there is a root of $f(r) = 0$ between 1 and 2	1A	Signs of $f(1)$, $f(2)$

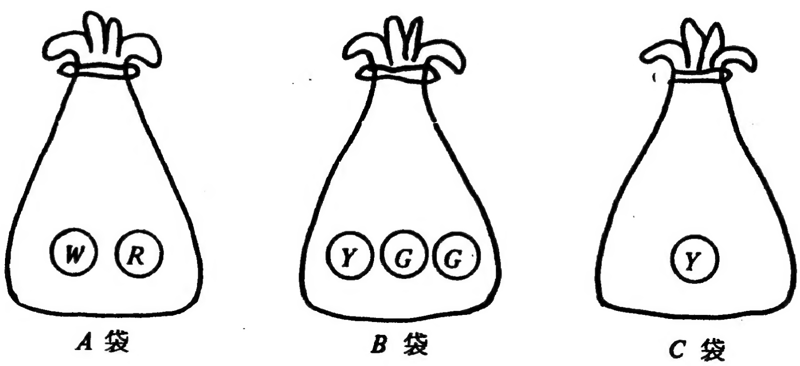
Interval	Mid -value, r_i	$f(r_i)$
$1 < r < 2$	1.5	−
$1.5 < r < 2$	1.75	−
$1.75 < r < 2$	1.875	+
$1.75 < r < 1.875$	1.8125 (1.813)	−
$1.8125 < r < 1.875$	1.84375 (1.844)	+
$\therefore 1.8125 < r < 1.84375$		

Solutions	Marks	Remarks
(bisection table above)	1M	Testing sign at mid-value
	1M	Choosing interval
$\therefore\ r = 1.8$ (correct to 1 d.p.)	1A	
	5	

Solutions	Marks	Remarks
12.(a) (i) The modal class is \$6000 – \$7000	1A	Accept \$6500
By symmetry of the distribution,		
the median salary = \$6500,	1A	
the mean salary = \$6500.	1A	
The interquartile range = 8000 – 5000	1A	Optional
= \$3000	1A	
The mean deviation		
$= \frac{1}{100} \times 2\,[7(6500-2500) + 8(6500-3500)$		
$+ 10(6500-4500) + 15(6500-5500)]$	1A	
= \$1740	1A	
	7	
(ii) The standard deviation of salaries will become smaller because the salaries of the additional 10 employees have no deviation from the mean while the total number of employees has become larger.	1A	For answer
	1	OR By calculation
		$\Sigma(x - x)^2$ unchanged Σf is greater
	2	
(b) The standard deviation		
$= \sqrt{\frac{1}{7}(9+4+1+0+1+4+9)}$	2A	
= 2	1A	
	3	



Solutions	Marks	Remarks
13.(a) (i) The probability that Bag B is chosen $= \frac{1}{3}$.	1A	
$\therefore$ the probability that the ball drawn is green		
$= \frac{1}{3} \times \frac{2}{3}$	1M	$P_1 \times P_2$
$= \frac{2}{9}$ (0.222)	1A	
(ii) the probability that Bag B is chosen and the		
yellow ball is drawn $= \frac{1}{3} \times \frac{1}{3}$	1A	OR probability of drawing Y from bag C.
$= \frac{1}{9}$ (0.111)		
$\therefore$ the required probability $= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times 1$	1M	$P_1 + P_2$ $\quad \frac{1}{3} \times \frac{1}{3} \times 1$ no mark
$= \frac{4}{9}$ (0.444)	1A	
	6	
(b) (i) The probability that Peter and Alice both draw		
a green ball $= \frac{2}{9} \times \frac{2}{9}$	1M	Followed from (a)(i)
$= \frac{4}{81}$ (0.0494)	1A	
(ii) The probability that they both draw a yellow		
ball from Bag B $= \frac{1}{9} \times \frac{1}{9}$	1A	
$= \frac{1}{81}$ (0.0123)		
The probability that they both draw a yellow ball		
from Bag C $= \frac{1}{3} \times \frac{1}{3}$	1A	
$= \frac{1}{9}$ (0.111)		
$\therefore$ the required probability $= \frac{1}{81} + \frac{1}{9}$	1A	
$= \frac{10}{81}$ (0.123)	~~2A~~ 1A	
	6	



Solutions	Marks	Remarks
14.(a) (i) The integers in G_6 are 16, 17, 18, 19, 20, 21	1M+1A	1M for 6 consecutive integers (5 correct)
(ii) The total number of integers in G_1 , G_2 , ..., G_6		
$= 1 + 2 + 3 + \ldots + 6$	1A	Optional
$= 21$	1A	
	4	
(b) (i) $u_{k-1} = 1 + 2 + \ldots + (k - 1)$	1A	
$= \frac{(k-1)}{2}[1 + (k - 1)]$	1M	Sum of AP $= \frac{n}{2}[a + \ell]$
$= \frac{k(k-1)}{2}$	1A	
$\therefore$ The first term in $G_k = \frac{k(k-1)}{2} + 1$ $\left(= \frac{k^2-k+2}{2}\right)$	1M+1A	1M for $v_1 = u_{k-1} + 1$
(ii) The sum of all integers in G_k		
$= \frac{k}{2}\left[2\left(\frac{k(k-1)}{2} + 1\right) + (k - 1) \times 1\right]$	1M+1A	1M for Sum of AP
$= \frac{k(k^2+1)}{2}$ $\left(= \frac{k^3+k}{2}\right)$	1A	
	8	